MATHEMATICAL MODEL FOR RELIABILITY ASSESSMENT OF DEVICE FOR PREPARATION AND DISTRIBUTION OF ANIMAL FEED AS “MAN-MACHINE”

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Abstract. Raising the reliability of machines for preparation and distribution of feed in animal husbandry is an urgent scientific and practical task. The purpose of the work is to develop a mathematical model of reliability of a technical system “Man-Machine” (using as an example machines for preparation and distribution animal feed) in order to identify the influence of its main components upon the dynamic characteristics with a gradual loss of its initial parameters. The model is described by Holmogorov’s stochastic differential balance equations of probabilities of states and transitions. The system of equations was solved in Laplace transformations. The probabilities of states in the form of transitions from originals to images were found according to Cramer’s rule. Under extreme conditions at the start of the operation and under the conditions of a settled pseudo-static mode during long-term operation of the systems, a readiness function was obtained that corresponds to the physical essence of the systems’ operable state. With an increase in the time of operation of the “man-machine” system its readiness for work under the conditions of “aging” of the equipment and increasing the operators’ fatigue decreases asymptotically, approaching its finishing value. It has been established that the optimal value of the probability of failure-free operation within the range of 0.92-0.98 corresponds to 350 - 500 hours of operation of the device for preparation and distributing feed. The probability of non-failure operation of the “human operator” has a higher value compared to the probability of non-failure operation of the “machine”. Within the indicated operating time the probability of failure-free operation is respectively: for the “machine” - 0.84-0.89; for the “human operator” - 0.93-0.98.

Keywords: system, reliability, machine, operator, failure, recovery.

Introduction

Raising the efficiency of animal husbandry is closely related to mechanized preparation and distribution of feed [1; 2]. Improvement of the technical and economic indicators of the technological process of preparation of feed mixtures, including grinding, mixing, transportation, distribution, dosing, provides for a set of innovations to improve the reliability of the respective machines. Measures to ensure the reliability of the means for preparation and distribution of feed should be taken, first of all, based on scientific research that is aimed at: analysis and synthesis of complex technical systems “Man-machine”; the use of the best practices; taking into account the operating conditions of the machines. In a number of recent years’ scientific investigations, there have been studied issues of increasing the efficiency of the use and improvement of the design of means for preparation and distribution of feed [1-5].

The most traditional equipment for the preparation of feed are machines, which consist of a hopper, grinding, mixing and distribution mechanisms [4-6]. Justification for the design parameters of the working bodies of means for the preparation and distribution feed, and the relative movement of the feed particles are considered in scientific articles [7-9]. In recent years a special place in many investigations [11-14] is devoted to scientific approaches to the assessment and ensuring the reliability of complex technical systems in the crop production, animal husbandry, and transport as complex technical systems “Man-Machine”. Research of the influence of individual components upon the reliability of complex technical systems is reflected in scientific articles [10, 11]. For this purpose, a mathematical model was presented in [10], based on a developed graph of states and transitions of a transport system, for the description of which Markov chains were used.

In investigation [11], to analyze the dangers of mechanized processes in agriculture, there are used Markov’s random processes with discrete states and continuous time. The article presents a labelled graph of the states of dangerous situations, intensities of the event flows are established. In article [12] there is reflected raised efficiency of systems in the logistics and transport services industry, based on the analysis of individual chains and links, optimization of the intensity of their transitions.
Investigations show that the intensity of transitions of complex technical systems to different states can be established, based on the use of statistical data [11], information on the accumulation of damage to parts from the results of flaw detection [13], diagnostics of the technical condition of machines [14], analysis of logistics systems [12]. Technical systems “Man-Machine”, which are based on close interaction of a human operator and a machine, are rather complex to study, and can physically be at different stages of functioning, starting from increasing the potential possibilities and ending with aging. We have proposed a methodology for calculating the reliability of the means for preparation and distribution of feed, based on the analysis of a labeled graph of states of functioning of the complex technical system “Man-Machine” [15].

Scientific and practical interest in the study of “Man-Machine” systems, using as an example for the preparation and distribution of feed, follows from the real situation, when the equipment gradually loses its reliability during its aging. At the same time the personnel that controls the machine, changes their attitude towards it, loses attention when fatigue accumulates, and the reaction decreases in the process of making the right management decisions, especially in possible non-standard situations that arise during the operation or maintenance of the equipment. Particularly important in scientific and practical terms is the study of the reliability of systems in non-stationary operating conditions in order to identify the quantitative characteristics and indicators, as well as the trends of their change.

The purpose of this work is to develop a mathematical model of reliability of a complex technical system “Man-Machine” in order to identify the influence of its main components upon the dynamic characteristics at a gradual loss of its initial parameters.

Materials and methods

The state of material and technical support of the livestock industry of Ukraine has been studied in the period from 2015 to 2019. It was established that the largest number of machines and mechanisms for the preparation and distribution of feed went to the livestock farms in 2016, and it amounted to 10722 machines. In 2019, the number of these vehicles decreased to 9086 units. The group of the most common manufacturers of machines and mechanisms for the preparation and distributing of feed in Ukraine includes both foreign (“Siloking”, “Trioliet”, “Kuhn”, “Strautmann”) and domestic agricultural machine plants (ODO “Braclav”, LLC “Demi-mix-Ukraine”, LLC “Umanfermmash-Agro”). The investigations were carried out during the operation of the machine “Braclav” for the preparation and distribution of feed mixtures for cattle (Fig. 1). The presented machine provided simultaneous grinding, mixing, transportation, dosing and distribution of the feed mixture.

![Fig. 1. General view of the “Braclav” machine for preparation and distribution of feed (a), and an auger with knives of the grinding-mixing mechanism (b) in the production](image-url)
and mathematical description of the behavior of complex technical systems, translating non-Markovian processes of its transitions into Markovian ones. The probabilities of states in the form of transitions from originals to images were obtained using Cramer's rule [16]. The basis of mathematical formalization in the research of this situation can be the construction of a labeled graph of states and transitions of the “Man-Machine” system to different possible states during operation (Fig. 2). It is appropriate to make the assumption that the complex technical system “Man-Machine” can be in two main states: operational – 0 and inoperable – 1 (elimination of failures, renewal, repair).

Fig. 2. Labeled graph of states and transitions of the “Man-Machine” system to various possible states: 0 – operable state; 1 – inoperable state (elimination of failures); (0'), (0'') – intermediate (fictitious states); \( \lambda'_0, \lambda''_0, \lambda'_1, \lambda''_1 \) – intensities that characterize the formation of failures, \( \mu \) – recovery intensity

Fictitious states – (0'), (0'') are introduced additionally to simplify the calculations of the mathematical description of the behavior of the system, translating the non-Markov processes of their transitions into Markov ones. It is envisaged that after each restoration the system acquires the same properties as a new one. The process of the system functioning is considered as a sequence of transitions between operable and inoperable states. In mathematical modelling of the description of the operation of a complex technical system “Man-Machine”, which is being researched, the following assumptions are made: – the flows of failures and restorations are simple Markov’s ones; – the system recovery begins immediately after its failure; – introduction of the system into operation occurs immediately after the completion of the process of its restoration; – the restored system is not inferior in its characteristics to the new one.

By introduction of additional fictitious states and the above assumptions, the model of functioning of the “Man-Machine” system can be represented by the following stochastic differential balance equations for the probabilities of states and transitions, using the Kolmogorov equations [16]:

\[
\begin{align*}
d\frac{d}{dt}P_0(t) &= -\lambda'_0 P_0(t) + \mu P_1(t) - \lambda''_0 P_0(t); \\
d\frac{d}{dt}P_1(t) &= \lambda'_1 P_0(t) - \mu P_1(t) + \lambda''_1 P_1(t); \\
d\frac{d}{dt}P_0'(t) &= -\lambda'_0 P_0'(t) + \lambda''_0 P_0(t); \\
d\frac{d}{dt}P_1'(t) &= -\lambda'_1 P_1'(t) + \lambda''_1 P_1(t).
\end{align*}
\]

where \( P_0(t) \) – probability of the system being in a working state;
\( P_0'(t) \) – probability of the system being in an intermediate (fictitious) state due to a decrease in the operator’s attention to the equipment;
\( P_0''(t) \) – probability of the system being in an intermediate (fictitious) state as a result of natural aging of the machine;
\( P_1'(t) \) – probability of the system being in an inoperable state.

The solution of the system of differential equations (1) involves the use of a normalized condition for the probabilities of states of an ergotic system with the following components:

\[
P_0(t) + P_1(t) + P_0'(t) + P_0''(t) = 1.
\]

For the initial condition we can take the situation when the components of the subsystems “man” and “machine” ensure the operation of the system, and it is in a working state. Thus, for the time \( t = 0 \),
the probability \( P_0(t) = 1 \). Accordingly, the probabilities of other states of the system are equal to zero: 
\( P_1(t) = 0; \ P_0'(t) = 0; \ P_0''(t) = 0 \). It is expedient to solve the system of equations (1) in the Laplace transforms [16]. Based on the adopted methodology and taking into account the given initial conditions, we can write:

\[
\begin{align*}
S\phi_0(S) - 1 &= -\lambda_0'\phi_0(S) + \mu\phi_1(S) - \lambda_0''\phi_0(S) = 1; \\
\phi_0(S) + \phi_1(S) + \phi_0(S) + \phi_0''(S) &= 1; \\
S\phi_0(S) - 0 &= -\lambda_0'\phi_0(S) + \lambda_0''\phi_0(S); \\
S\phi_0''(S) - 0 &= -\lambda_0'\phi_0''(S) + \lambda_0''\phi_0(S).
\end{align*}
\]

After the performed transformations we obtain a system of algebraic equations. The coefficients of the unknowns form the following matrix (4).

\[
\Delta = \begin{bmatrix} (S + \lambda_0' + \lambda_0'') - \mu & 0 & \lambda_0'' \\ S & S & S \\ -\lambda_0' & 0 & S + \lambda_0' \\ -\lambda_0'' & 0 & (S + \lambda_0') \end{bmatrix}
\]

(4)

Achievement of the pursued goal of the research is considered finding the probabilities of the states of the ergotic system in order to determine the corresponding indicators of reliability. The probabilities of states in the form of transitions from originals to images can be found according to Cramer’s rule:

\[
\phi_0(S) = \frac{\Delta_i}{\Delta}.
\]

(5)

It is evident from the presented relationship that, in order to establish any of the reliability indicators and find probabilities \( \phi_0(S) \) for this, it is necessary to have a solution to the main determinant (5). A rational way to solve the main determinant is to lower its order initially. We carry out such lowering by disintegration of the expression around the first line. Based on this, it seems possible after a series of transformations to obtain:

\[
\Delta = S^4 + S^3(\lambda_0' + \lambda_0'' + \lambda_0' + \lambda_0'' + \mu) + S^2(\lambda_0'\lambda_0'' + \lambda_0'\lambda_0'' + \lambda_0'\lambda_0'' + \lambda_0'\lambda_0'' + \mu\lambda_0' + \mu\lambda_0'' + \mu\lambda_0' + \mu\lambda_0'') + \\
+ S(\mu\lambda_0'\lambda_0'' + \mu\lambda_0'\lambda_0'' + \mu\lambda_0'\lambda_0'') + (6)
\]

To determine the value of the determinant \( \Delta \), we can take out the common factor \( S \) and introduce the replacement into formula (6):

\[
K = \lambda_0' + \lambda_0'' + \lambda_0' + \lambda_0'', \\
P = \lambda_0'\lambda_0' + \lambda_0'\lambda_0'' + \lambda_0'\lambda_0'' + \lambda_0'\lambda_0'' + \mu\lambda_0' + \mu\lambda_0'' + \mu\lambda_0' + \mu\lambda_0'', \\
N = \lambda_0'\lambda_0'\lambda_0'' + \lambda_0'\lambda_0'\lambda_0'' + \lambda_0'\lambda_0'\lambda_0'' + \lambda_0'\lambda_0'\lambda_0''.
\]

(7)

Results and discussion

Using the previously obtained expressions for the constant values of \( K, P \) and \( N \) (7), formula (6) can be presented in the following form:

\[
\Delta = S\left[S^3 + KS^2 + PS + N\right].
\]

(8)

After a series of transformations, we obtain the following dependence:

\[
\phi = \frac{K}{S} + \frac{P}{S - S_3} + \frac{N}{S - S_3} + \frac{Q}{S - S_4} = \\
S^2\left[(K + P)S_3 + (K + P)S_3 + NS_3 + QS_3\right] + S\left[(K + P)S_3 + S_4\right].
\]

(9)
To solve the system of the obtained equations, we use the method of successive substitutions. By the inverse Laplace transformation the complex technical system “Man-Machine” can be presented as follows:

\[ P_0(t) = K_x(t) = K \exp(-S_1t) + P \exp(-S_2t) + N \exp(-S_3t) + Q \exp(-S_4t). \] (10)

The obtained dependence (10) can be checked by evaluating the features of its components. To establish dependence only on the rates of failures and restorations, we will carry out exchange of roots. Then, using the abbreviated multiplication formula, after the corresponding mathematical transformations, in the final form we obtain a dependence:

\[ P_0(t) = K_x(t) = \frac{\lambda_1^2 \lambda_2^2 + \lambda_1^0 \mu + \lambda_1^1 \mu}{\lambda_1^2 \lambda_2^2 + \lambda_1^0 \lambda_2^0 + \lambda_1^1 \lambda_2^0 + \lambda_1^2 \lambda_2^0 + \lambda_1^0 \mu + \lambda_1^1 \mu}. \] (11)

The above analysis, taking into account the exponential components (10), opens up a possibility of constructing graphic dependences of the change of reliability upon the time of operation of complex technical systems “Man-Machine” (Fig. 3). There is presented a possibility of a quantity assessment of the loss of performance of a complex technical system “Man-Machine” with a decrease of its potential reliability as a result of equipment aging and operator’s fatigue. Investigations of the dynamics of changes in the probability of failure-free operation showed that the value of the noted reliability indicator depends on the influence of the “machine” and “human operator” components. Based on the research, it was established (Fig. 3a) that the optimal value of the probability of failure-free operation within 0.92-0.98 corresponds to an intensity of 0.002-0.003 h⁻¹, which will correspond to 350-500 hours of operation for preparation and distribution of feed.

![Graph 1](image1.png)

![Graph 2](image2.png)

**Fig. 3.** Dependencies: a) - probabilities of trouble-free operation of the machine for preparation and distribution feed; b) - probabilities of failure-free operation from the failure and recovery rates of the “machine” and “human operator” components
Analysis of the graphic dependencies, which are shown in Fig. 3b, shows that probability of a non-failure operation of the “human operator” has a higher value compared to the probability of a non-failure operation of the “machine”. Within the operating time of 300-500 hours, the probability of a failure-free operation is respectively: for the “machine” – 0.84-0.89; for the “human operator” – 0.93-0.98.

Conclusions
1. The model of the functioning of the “Man-Machine” system can be presented by stochastic differential equations for the balance of the probabilities of states and transitions, using the Kolmogorov equations. For preparation and distribution of feed it gradually loses its efficiency as a result of “aging” of its elements and assemblies, as well as the loss of the operator’s efficiency level because of increased fatigue, reduced attention and skill when controlling the complex equipment.
2. The conditions for the operation of a complex technical system “Man-Machine” are reduced to Markov flows of distributing the time of work and recovery by introducing additional fictitious states. The main determinant of the system of equations, which describes the transitions of a complex technical system to different possible states, includes a combination of $\lambda$, $\mu$ characteristics, and is a necessary calculation element for determination of the dynamic characteristics of reliability.
3. It has been established that the optimal value of the probability of failure-free operation in the range of 0.92-0.98 corresponds to 350-500 hours of operation for preparation and distribution of feed. The probability of non-failure operation of the “human operator” has a higher value, compared to the probability of non-failure operation of the “machine”, and within the indicated operating time, the probability of non-failure operation, respectively, is: for the “machine” – 0.84-0.89; for the “human operator” – 0.93-0.98.

References

[12] Bulgakov V., Pascuzzi S., Beloev H., Ivanovs S. Theoretical investigations of the headland turning agility of a trailed asymmetric implement-and-tractor aggregate. Agriculture (Switzerland), Vol. 9 (10), 2019, art. 224


