

## ENGINEERS' MATHEMATICS EDUCATION IN THE CONTEXT OF SUSTAINABLE DEVELOPMENT

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**Abstract.** The article is the result of scientific analysis and assessment of scientific literature and a number of information sources taking into consideration the authors' reflection experience and observations in connection with the development of the curriculum in mathematics for engineers. It includes several factors determining the mathematics role in sustainable development as well as in engineering education and provides insight into "A Framework for Mathematics Curricula in Engineering" created by the European Society for Engineering Education Mathematics Working group. Eight competences of mathematical education of engineers are outlined in the study. The first four competences include abilities to ask and answer questions in and with mathematics as a result of mathematics for engineers. The next four competences reflect the ability to deal with and manage mathematical language and tools. The study in the context of sustainable development of society outlines the role of mathematics in three aspects: social, environmental, and economic. The study also covers mathematical challenges for sustainability and actions important for effective doing. The aim of the study was to outline the necessary mathematical competences for engineers, exploring the role of mathematics in sustainable development and in engineering education as well as to identify challenges of mathematics for sustainability and criteria or dimensions for specifying to which extent students should have obtained it at a certain stage of their mathematical education.

**Keywords:** engineering, competence, mathematics, mathematical competence, sustainable development.

### Introduction

Science (including mathematics) is a basic element in modern education. It is indispensable in all areas of life – everyday, social and professional life. In various specialties the science and mathematical competence is an essential element. Traditionally, mathematics teaching goal at universities has been to give knowledge for the best learning of technical subjects. Of late years the main objective of mathematics at the Latvia University of Agriculture (LLU) was revised describing the learning outcomes in accordance with the labour market needs as well as taking into account the development of information technologies. So, the improvement of the programme of mathematics was based on two categories of description of the experience of mathematics: mathematics as the basis of other subjects and as a tool for analysing problems that occur in the world.

There are some important factors that affect the development of mathematics education for engineers in Latvia: despite the fact that the size of general engineering studies in the authors' university by the means of contents has not been changed for several years, it tends to decrease by the means of time. Moreover, the problem is that the curriculum has evolved through addition not redesign. At present, curriculum in mathematics is specified by means of four components: the course aim; specification of mathematical content – lists of mathematical topics; results to be covered; forms and instruments of assessment and testing.

Nowadays, the objectives of mathematics can be outlined as follows: graduates have to be prepared to act in a sustainable manner. At first, meaning of the term "engineering" is currently changing. Nowadays, the latest findings define engineering as a creation of something new by using science and mathematics. For example, the Accreditation Board of Engineering and Technology [1] defines engineering as a profession connected with studies of mathematics and natural sciences, experience and practice that is applied to develop ways how to use economically the natural resources to facilitate the existence of the mankind. The other factors influencing engineering education is increasing complexity, because engineering projects, products and systems are becoming more complex. The second factor is sustainability which becomes an important part of engineering practice because we cannot ignore the problem of climate change and global warming. Sustainability requires a critical consideration of the development of all engineering products [2].

The concept of sustainable development is based on the need to optimise the development of economic and social systems, as well as the impact on the environment and resources. The three spheres, without which modern mankind is not possible, have a functioning economy, harmonious

society and a healthy environment, at the same time external preconditions for an individual's development are desirable. Sustainable development means that any economic, social or environmental issue to be addressed so that the decision would be favourable or unfavourable as little as possible for the development of the rest of the sphere.

To act in a sustainable manner, it is necessary for the appropriate knowledge, skills and ability to live responsibly and prosperous in a limited world. Therefore, one of the factors that allows society a long-term and successful development, is education. It contributes to every member of the society and, in particular young people, active, creative and productive attitude and development of the required competences.

It means that the main task of universities is to provide the new specialists with the necessary knowledge and skills, which are related to work in changing conditions. That is why more than ever engineering education is challenged to make deep changes in the programs and structures. That poses new challenges for mathematics and outlines new directions for future development of mathematics education for engineers also at the LLU. For that reason it is important to define the role of mathematics studies in engineering education and realize the necessary changes in mathematics study programmes, which could be divided into two directions: the curriculum development as well as the organization of the study process.

The aim of the study was to outline the necessary mathematical competences for engineers, exploring the role of mathematics in sustainable development and in engineering education as well as to identify challenges of mathematics for sustainability and criteria or dimensions for specifying to which extent students should have obtained it at a certain stage of their mathematical education.

## **Materials and methods**

The problem has been approached by analysing and evaluating the scientific literature as well as a number of information sources and reports, taking into consideration the authors' reflection experience and observations.

The article includes theoretical analysis of topical issues like the definitions of sustainable development, education for sustainable development, mathematical competence for engineers as well as theoretical framework for mathematics curriculum in engineering education.

In the UN General Assembly Report of the World Commission on Environment and Development "Our Common Future" [3] sustainable development is defined as an ongoing, deliberate and logical managed social change process at the world, regional and local level, the aim of which is to provide human needs of the present, without compromising the ability of future generations to meet their own needs. The concept of sustainable development is not just an idea of the development of humanity in general or the particular community, but mostly the view set, to ensure the society existence.

In Latvia, education for sustainable development (ESD) is defined as education that contributes to each individual's opportunities to acquire knowledge, values and skills needed for participation in decision-making on an individual or collective action at local and global level to improve the quality of life now, without compromising future generations [4].

Integration of sustainable development into an educational system at all levels is an important challenge that has been encountered in recent years. Universities have a special responsibility to apply their intellectual resources to identify, verify and promote policies, mechanisms and procedures, which lead to sustainability in all aspects of human endeavour. This means that the universities have to ensure sustainable development in their curricula, teaching practices, research and consulting, community service activities, institutional practices, promoting the achievement of sustainable futures embracing ecological, economic and social aspects of human existence [5]. A particular attention should be paid to engineering programmes.

In each university programme the knowledge, skills and values necessary for sustainable development of society have to be included. The Engineering Council has defined the role of professional engineers in sustainability using the following six principles [6]: contribute to building a sustainable society, present and future, apply professional and responsible judgement and take a leadership role, do more than just comply with legislation and codes, use resources efficiently and effectively, seek multiple views to solve sustainability challenges and manage risk to minimise

adverse impact on people or environment. In keeping with these principles, a primary objective of higher education engineering curricula is to produce graduates to be able to undertake engineering activities in a way that contributes to sustainable development. It requires understanding of the natural world and relationships with it. To acquire that understanding, progress in the mathematical sciences is essential.

Mathematics role in sustainable development is characterized by a number of factors, including, mathematical competence compliance with competencies for sustainable development. A number of research studies have been done on knowledge, skills and competences in relation to sustainable development. The concept of competences is seen as an essential landmark for orienting teaching and learning for sustainable development [7-9]. Different competences as a fundamental element of sustainable development have been widely discussed by several authors:

1. problem solving, critical thinking, action competence and systems thinking [10; 11];
2. imagination, critical thinking and reflection, systemic thinking, partnerships, learning to work together, participation in decision-making [12];
3. systems thinking – the ability to see the interconnections between different dimensions and the complexity of systems and situations [13; 14].

Apparently education for sustainable development provides that any person acquires fundamentally other solutions to various problems. This requires independence judgments, open and flexible thinking, willingness to critically evaluate new information, the ability to distinguish opinions from facts readiness to re-evaluate their assumptions and not to take solutions as final and unalterable. In turn, the EU Directive states that “mathematical competence” is based on the ability to solve everyday problems using models of thinking (logical or spatial), representation (formulas, design, graphs, charts, etc.). It includes the ability to identify the structure and commitment, repetition or regularity [15]. Thus, in context of sustainable development of society mathematics has an important role in all its aspects: social, environmental and economic. First, mathematics provides an understanding of the world and its regularities (maths as approach to life). Second, mathematics is a tool to describe and solve the problems facing us, it provides us with the tools to make informed decisions (maths as a technic or component). Third, mathematics provides a direct contribution to sustainable development, with a variety of mathematical models for the help in the planning of resource recovery processes, controlling or reducing the possible consequences (maths as models).

Mathematics role in sustainable development may also be characterized by coherence between mathematical competence and capacity which engineers should have. By a mathematical competence is understood an individual's insight-based capability to purposefully and successfully deal with situations that (re)present a particular kind of mathematics-laden challenge. According to M. Niss [16; 17] to possess a competence (to be competent) in some domain of personal, professional or social life is to master essential aspects of life in that domain. Mathematical competence then means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role. Necessary, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills [17]. In turn, a number of studies of engineering education in the context of sustainable development determine that engineering students regarding sustainable development should learn [18]: (1) what the problems are – the capacity to analyse the problems at different scales, with a systems approach – providing solutions is considered to be more important than analysing problems; (2) how to solve them – methods to develop technologies that could contribute to solutions - students should learn to think in longer term processes and define their work within such longer term processes that cannot be fully controlled.

## Results and discussion

It is well known that the level of social and economic development of any country is intimately connected with the level of development of that country in science and technology. Since mathematics is known to be at the foundation of science and technology, it means that the level of social and economic development is closely connected with the level of development in the mathematical sciences. It should be noted, that no society can develop without effective teaching and learning of

mathematics in schools. A gap in the level of development between the advanced countries and the developing countries is as a result of the gap in the level of teaching and learning of mathematics [19].

Apparently mathematics plays a big role in sustainable development in all of its aspects: social, environmental, and economic. Mathematical science challenges are enormous (Table 1). Sustainability issues are hugely complex, requiring subtle scientific and mathematical tools [20].

Table 1

### Mathematical challenges for sustainability

Challenge	Characteristics of challenge	Actions important for effective doing
Human well-being and the natural environment	Focused on the interrelationship between human needs and ecological needs. Two ways to act sustainably: 1. To use resources not more quickly than nature can regenerate them; 2. To deplete natural stocks and to convert them into another form of capital at a rate that is capable of maintaining human well-being over the long term.	1. Precisely quantify natural capital as well as human and natural well-being; 2. Understand how people activities affect natural capital; 3. Calculate how quickly nature can regenerate; 4. Develop ways people can adapt to a changing environment; 5. Make responsible decisions balancing the needs of people today with the needs of future generations and balancing the needs of different people around the world.
Human-Environment Systems as Complex Adaptive Systems	The interactions between humans and the environment are both extraordinarily complex and constantly changing, with interacting feedbacks between different parts of the system.	How mathematics of complex adaptive systems can illuminate the interactions between humans and their environment.
Measuring and monitoring progress toward sustainability	To learn to live sustainably, people have got to know how well they are doing.	1. Collect a huge amount of data; 2. Get the most information possible given limited resources; 3. Make sense of the data got.
Managing Human-Environment Systems for Sustainability	The central point is to guide decision – making.	Maths tools needed to put together what we know into a precisely defined set of questions and into a practical course of action.
Mathematical challenges in energy sustainability	The energy system needs a radical transformation, fast, and so does the relation of human activity to energy.	An in-depth case study that touches on all four previous groups.

The mathematical education is important for a high quality engineering education. There is no doubt that working with mathematical models and solving problems using mathematical procedures form an essential part of engineering work. Mathematical education for engineers is determined by the documents worked out by the European Society for Engineering Education Mathematics Working Group. Three curricula documents focused on a target group with a few typical features are outlined in the article: A core curriculum in mathematics for the European engineer [21] – contains a list of topics to be covered in any engineering degree programme; Mathematics for the European Engineer – A Curriculum for the Twenty-First Century [22] – resulted in a quite detailed organized list of content-related learning outcomes; A Framework for Mathematics Curricula in Engineering [23] – used the concept “mathematical competence” to specify the higher-level learning goals and emphasized what students should be able to do as a result of the learning process.

The main purpose of “A Framework for Mathematics Curricula in Engineering” [23] is to provide orientation for those who set up a concrete mathematics curriculum for a specific engineering

programme. So the concept of mathematical competence is based on the Danish KOM(Competencies and the Learning of Mathematics) project, which set up eight competences. The first four competences include abilities to ask and answer questions in and with mathematics as a result of mathematics for engineers:

1. Thinking mathematically: knowledge of the kind of questions that are dealt with in maths and the types of answers maths can and cannot provide, and the ability to pose such questions; recognition of maths concepts and understanding of their scope and limitations, extending the scope by abstraction and generalisation of results; understanding of the certainty of mathematical considerations;
2. Reasoning mathematically: ability to understand and assess an already existing maths argumentation and the notion of proof, and to recognise the central ideas in proofs; knowledge/ability to distinguish between different kinds of maths statements; construction of chains of logical arguments and hence of transforming heuristic reasoning into own proofs (reasoning logically);
3. Posing and solving mathematical problems: ability to identify and specify mathematical problems; ability to solve mathematical problems (including knowledge of the adequate algorithms); personal capabilities to decide a question considered as a problem;
4. Modelling mathematically: ability to analyse and work in existing models; ability to perform active modelling.
5. The next four competences reflect the ability to deal with and manage mathematical language and tools:
6. Representing mathematical entities: ability to understand and use mathematical representations and know their relations, advantages and limitations; ability to choose and switch between representations;
7. Handling mathematical symbols and formalism: ability to understand symbolic and formal maths language and its relation to natural language as well as the translation between both; rules of formal maths systems and the ability to use and manipulate symbolic statements and expressions according to the rules;
8. Communicating in, with, and about maths: ability to understand mathematical statements (oral, written or other) made by others; ability to express oneself mathematically in different ways;
9. Making use of aids and tools: knowledge about the aids and tools that are available; Ability to use the aids and tools thoughtfully and efficiently.

The third edition of the curriculum document provides also criteria or dimensions for specifying to which extent students should have obtained it at a certain stage of their mathematical education: (1) Degree of coverage – is the extent to which the person masters the characteristic aspects of a competence; (2) Radius of action – comprises the contexts and situations in which a person can activate a competency; (3) Technical level – indicates how conceptually and technically advanced the entities and tools are with which the person can activate the competence.

To specify the desired degree of coverage, it is suggested to use “clusters” described in the OECD PISA Assessment Framework [24]: (1) Reproduction cluster – comprises tasks where students are required to recall or reproduce facts, formulations, procedures, manipulations, tool usage patterns etc. learned and practiced before in the educational process in familiar contexts and situations; (2) Connections cluster - consists of abilities where students have to connect knowledge acquired before or they have to apply it in situations and contexts, which are at least slightly different from those where they first used it. It also includes the handling of more complex argumentations and procedures where complexity results from connecting several steps of argumentation or computation; (3) Reflection cluster is concerned with abilities where students have to apply mathematics in new contexts and situations, so they have to reflect on which mathematical concepts to use and to combine, how to formulate a mathematical problem and how to combine existing and develop new concepts in order to solve the given problems.

Mathematical competences are meant in the Danish KOM project to be the same at any educational level. That is why they cannot be employed to determine the mathematical content/topics to be on the agenda in a given educational context. Developing new engineering mathematics programs the relationship between mathematical contents/topics and competencies is considerable.

According to M. Niss [17] competences and mathematical topic areas are viewed as orthogonal – how the corresponding competence manifests itself when dealing with the corresponding topic at the educational level at issue. The degree, to which the eight competencies have to be covered for sustainable development, also have to be specified.

In “A Framework for Mathematics Curricula in Engineering” [23] also the described content-related competences (learning outcomes) dealing with knowledge and skills are identified for engineering education. The content-related learning outcomes are arranged in a structure, which has four levels. These levels represent an attempt to reflect the hierarchical structure of mathematics and the way in which mathematics can be linked to real applications of ever-greater sophistication as the student progresses through the engineering degree programme.

In order to implement competence-based curriculum, in curriculum document the environment, which might help students to obtain the competencies to an adequate level, is discussed. It offers the orientation for designing teaching processes, teaching and learning environment and environment, and preferred approaches. The curriculum document includes the suitability of different teaching and learning arrangements like lectures, e-learning scenarios, tutorials or projects; the use of mathematics technology, which is ubiquitous in engineering and engineering education, mostly in the form of mathematics or application programs; integration of mathematics curriculum into an engineering study course; attitude of students towards the value of mathematics for their field of study and the consequences for their learning behaviour. The assessment issues are also discussed there.

## Conclusions

1. Mathematics plays a big role in sustainable development in all of its aspects: social, environmental, and economic. Thus, mathematics has to face several challenges and to act sustainable; there are several mathematical methods that are needed to do effectively.
2. Mathematics role in sustainable development is characterized by a number of factors, including, mathematical competence compliance with competencies for sustainable development as well as by coherence between mathematical competence and capacity which engineers should have.
3. “A Framework for Mathematics Curricula in Engineering” [23] provides orientation for setting up concrete mathematics curriculum for specific engineering programs. There are:
  - eight mathematical competences for engineers to be covered for sustainable development as well as three criteria/dimensions of its progress;
  - content-related competencies (learning outcomes) dealing with knowledge and skills, which have four levels;
  - outlined teaching and learning environment, which might help students obtain the competences to an adequate level as well as assessment issues.
4. Developing new engineering mathematics programmes have to be taken into account:
  - relationship between mathematical contents/topics and competences, specifying how the corresponding competence manifests itself when dealing with the corresponding topic at the educational level at issue;
  - the degree, to which the eight competencies have to be covered for sustainable development, also has to be specified;
  - teaching and learning environment, which might help students obtain the competences to an adequate level;
  - to see assessment as a part of the learning process that gives answers to questions what the students will learn and what they have to do to learn, and how the students’ knowledge has to be evaluated.

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