VISUALIZATION OF THE KINEMATICS OF A MATERIAL POINT

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Abstract. The reader of this paper will be acquainted with the experiences of visualization of engineering mechanics on the computer screen. The present paper introduces the method of creating a video clip by using the worksheet of the Computer Package Mathcad in order to visualize the motion of a material point on its trajectory, the change of the circle of curvature, the directions of vectors of velocity and vectors of acceleration and the formation of evolute of the trajectory. The results of this paper are illustrated by the video clip and by the figures. Mathcad worksheet as well as already composed video clips can be used by students and teachers of engineering mechanics.

Keywords: visualization, engineering mechanics.

Introduction

Due to the advances in computer technologies there have emerged many new ways to solve engineering problems, the solution of which requires correct equations and calculation methods, which may be unique in each case. Visualization of the studies of mechanical systems on the computer screen accelerates the process of getting correct solutions and thereby reducing the time spent and costs incurred with regard to product development.

Currently there are several computer programs that can create virtual mechanical models, most common of them are Matlab, Mathematica, Maple, Mathcad, etc. In the environment of the Computer Package Matlab we can carry out the studies of virtual models of manipulators with multiple degrees of freedom [1], the study of robot motions [2], the study of the industrial spinning process [3], the visualization of four degrees of freedom parallelogram bipedal robot walking [4]. The Computer Package Mathematica has been used for composing virtual models of four-bar linkage [5], and for the study of the car steering mechanism [5] and vibrations [6].

The authors of the present paper have chosen Computer Package Mathcad [7] for research and teaching purposes, because it is permanently used in the Estonian e-studies community at the universities and vocational schools http://www.e-ope.ee/. The developer is periodically updating and upgrading Mathcad in the server of this community.

Mathcad was used to study and visualize the motion of a virtual linkage [8], the motion of the free-active circular link on the top of a potato ridge [9], the working process of the virtual disk-ridging tool of a wide row potato field tillage machine [10], the working process of the virtual model of a blueberry harvester picking reel [11], the distribution of loads on helicopter blades [12], the working process of a novel virtual manipulator for a stone protector of a stony soil tillage implement [13], the motion of fertiliser granules on the plane of the spinning spreading disc of a disc spreader [14], the chaotic motion of virtual double pendulum [15], etc.

There has been composed a review of studies of a virtual manipulator for a scraper of a manure press removal [16].

Visualization of mathematics, mechanics and other engineering subjects allows the engineering students to improve their understanding of these subjects. There has been described an attempt to use Mathcad in teaching of engineering mathematics [17]. Mathcad has been used for composing several e-courses on engineering mathematics and mechanics, available on the website of repository http://www.e-ope.ee/repositoorium of Estonia e-University. A proposal has been made to use the Mathcad in creative teaching of engineering mechanics [18]. Also the use of Mathcad in the teaching and learning of analytical mechanics has been described [19].

This paper shows how to create, on the Mathcad worksheet, video clips visualizing the motion of a material point on its trajectory, the change of the circle of curvature, the directions of the vectors of velocity and the vectors of acceleration and the formation of evolute of the trajectory. The Mathcad worksheet as well as the already composed video clips can be used by students and teachers of engineering mechanics.

Basic formulas

If the curve is represented by the parametric equations $x = x(t)$, $y = y(t)$, then the radius of curvature at the point M $(x(t), y(t))$ is [20]

$$R(t) = \frac{v(t)^3}{\left|a_y(t) \cdot v_x(t) - a_x(t) \cdot v_y(t)\right|} \tag{1}$$

The centre of curvature co-ordinates x and y are $\alpha(t)$ and $\beta(t)$,respectively

$$\begin{aligned} \alpha(t) &= x(t) - \frac{v(t)^2 \cdot v_y(t)}{a_y(t) \cdot v_x(t) - v_y(t) \cdot a_x(t)}, \\ \beta(t) &= y(t) + \frac{v(t)^2 \cdot v_x(t)}{a_y(t) \cdot v_x(t) - v_y(t) \cdot a_x(t)}. \end{aligned} \tag{2}$$

In (1) and (2)

$$v_x(t) = \frac{d}{dt} x(t), v_y(t) = \frac{d}{dt} y(t) \tag{3}$$

$$a_x(t) = \frac{d^2}{dt} x(t), a_y(t) = \frac{d^2}{dt} y(t) \tag{4}$$

$$v(t) = \sqrt{v_x(t)^2 + v_y(t)^2} \tag{5}$$

Let us suppose now that a material point M is moving according to the law

$$\begin{aligned} x(t) &= A \cdot \cos(\omega \cdot t) \\ y(t) &= B \cdot \sin(\omega \cdot t) \end{aligned} \tag{6}$$

where a, b and ω are constants and t is the time.

The formulas (6) determine the trajectory of the point M. The formulas (3) and (4) determine the projections of velocity and acceleration of the point M on the x – and y – co-ordinate axes of the co-ordinate system O_{xy}. The formulas (5) represent the modules of the vectors of velocity of the point M.

The direction angles between the x-axis and vectors of velocity $\overrightarrow{v(t)}$ and acceleration $\overrightarrow{a(t)}$ are

$$\begin{aligned} \alpha_v(t) &= angle(v_x(t), v_y(t)) \\ \alpha_a(t) &= angle(a_x(t), a_y(t)) \end{aligned}$$

where $angle(x, y)$ returns on the worksheet of Mathcad in the direction angle (in radians) of a vector.

The direction angle of the vector, directed from the point M to the centre of curvature, is

$$\alpha_p(t) = angle(\alpha(t) - x(t), \beta(t) - y(t))$$

where

$$a(t) = \sqrt{a_x(t)^2 + a_y(t)^2}$$

The circle of curvature is determined by the following parametric equations

$$\begin{aligned} x_k(t) &= \alpha(t) + R(t) \cdot \cos(q) \\ y_k(t) &= \beta(t) + R(t) \cdot \sin(q) \end{aligned}$$

where $q = 0, 1, 2,\ldots 2\pi$. The tangential acceleration is defined by the formula [22]

$$a_\tau(t) = \frac{a_x(t)\cdot v_x(t) + a_y(t)\cdot v_y(t)}{v(t)} \tag{7}$$

and the normal acceleration is defined by the formula

$$a_n(t) = a_x(t)\cdot\frac{(\alpha(t) - x(t))}{R(t)} + a_y(t)\cdot\frac{(\beta(t) - y(t))}{R(t)}. \tag{8}$$

The columns $F(t, \gamma, b, n)^{(1)}$ and $F(t, \gamma, b, n)^{(2)}$ of matrix function

$$F(t,\gamma\ ,b,n) := \left|\begin{array}{l} x \leftarrow x(t) \\ y \leftarrow y(t) \\ V_0 \leftarrow \begin{pmatrix} x & x & x & x & x & x \\ y & y & y & y & y & y \end{pmatrix}^T \\ I \leftarrow \begin{pmatrix} 0 & 0.85 & 0.85 & 1 & 0.85 & 0.85 \\ 0 & 0 & 0.03 & 0 & -0.03 & 0 \end{pmatrix}^T \\ \Omega \leftarrow \begin{pmatrix} cos(\gamma\ (t)) & sin(\gamma\ (t)) \\ -sin(\gamma\ (t)) & cos(\gamma\ (t)) \end{pmatrix} \\ V_0 + n\cdot b(t)\cdot I\cdot\Omega \end{array}\right.$$

of Mathcad [21] can be used to simulate the movement of a geometric vector $\vec{b}$ on Mathcad worksheet. In the function $F(t, \gamma, b, n)$ the following notations are used: $x = x(t)$ and $y = y(t)$ are the co-ordinates of the origin, $\gamma = \gamma\ (t)$ is the direction angle, $b(t)$ is the module or projection (if the vector $\vec{b}$ is determined only by one co-ordinate), and b is notation of $b(t)$ of a vector $\vec{b}$; m is the coefficient of dimensions and I is the matrix of the shape of a vector $\vec{b}$ on Mathcad worksheet; Ω is the matrix of transformation of rotation, V_0 is the matrix of origin, T denotes the transposed matrix, t is the time.

At the fixed moment of time $t = T$ the columns of the matrix function $F(t, \gamma, b, n)$

$$V_v(T)^{(0)} = F(T,\alpha_v,v,0.2)^{(0)}, V_v(T)^{(1)} = F(T,\alpha_v,v,0.2)^{(1)},$$

where v is the notation of module $\sqrt{v_x(t)^2 + v_y(t)^2}$, visualize the vector of velocity, the columns

$$V_a(T)^{(0)} = F(T,\alpha_a,v,0.1)^{(0)}, V_a(T)^{(1)} = F(T,\alpha_a,v,0.1)^{(1)},$$

where a is the notation of module $\sqrt{a_x(t)^2 + a_y(t)^2}$ visualize the vector of acceleration, the columns

$$V_{an}(T)^{(0)} = F(T,\alpha_p,a_n,0.1)^{(0)}, V_{an}(T)^{(1)} = F(T,\alpha_p,a_n,0.1)^{(1)},$$

where a_n is determined by the formula (8), visualize the vector of normal acceleration, the columns

$$V_{a\tau}(T)^{(0)} = F(T,\alpha_v,a_\tau,0.1)^{(0)}, V_{an}(T)^{(1)} = F(T,\alpha_v,a_\tau,0.1)^{(1)},$$

where a_τ is determined by the formula (7), visualize the vector of tangential acceleration, the columns

$$V_{ap}(T)^{(0)} = F(T,\alpha_p,R,1)^{(0)}, V_{ap}(T)^{(1)} = F(T,\alpha_p,R,1)^{(1)},$$

where R is determined by the formula (1), visualize the vector that begins from the point M and is directed towards the centre of curvature (Fig. 1 and Fig. 2).

Results

Let us suppose, that $T = 1.7$ in (6) $A = 3$, $B = 2$, $\omega = \pi$. Then the defined columns are

$$V_v(T)^{(0)} = \begin{bmatrix} 1.763 \\ 3.060 \\ 3.037 \\ 3.288 \\ 3.082 \\ 3.060 \end{bmatrix}, V_v(T)^{(1)} = \begin{bmatrix} -1.618 \\ -0.990 \\ -0.944 \\ -0.879 \\ -1.036 \\ -0.990 \end{bmatrix}, V_a(T)^{(0)} = \begin{bmatrix} 1.763 \\ 0.284 \\ 0.236 \\ 0.023 \\ 0.332 \\ 0.284 \end{bmatrix}, V_a(T)^{(1)} = \begin{bmatrix} -1.618 \\ -0.261 \\ -0.313 \\ -0.021 \\ -0.208 \\ -0.261 \end{bmatrix},$$

$$V_{an}(T)^{(0)} = \begin{bmatrix} 1.763 \\ 0.950 \\ 0.890 \\ 0.806 \\ 1.009 \\ 0.950 \end{bmatrix}, V_{an}(T)^{(1)} = \begin{bmatrix} -1.618 \\ 0.062 \\ 0.033 \\ 0.358 \\ 0.091 \\ -0.062 \end{bmatrix}, V_{a\tau}(T)^{(0)} = \begin{bmatrix} 1.763 \\ 1.098 \\ 1.109 \\ 0.980 \\ 1.086 \\ 1.098 \end{bmatrix}, V_{a\tau}(T)^{(1)} = \begin{bmatrix} -1.618 \\ -1.940 \\ -1.964 \\ -1.997 \\ -1.917 \\ -1.940 \end{bmatrix},$$

$$V_{ap}(T)^{(0)} = \begin{bmatrix} 1.763 \\ 0.552 \\ 0.464 \\ 0.338 \\ 0.640 \\ 0.552 \end{bmatrix}, V_{ap}(T)^{(1)} = \begin{bmatrix} -1.618 \\ 0.883 \\ 0.840 \\ 1.324 \\ 0.925 \\ -0.883 \end{bmatrix}.$$

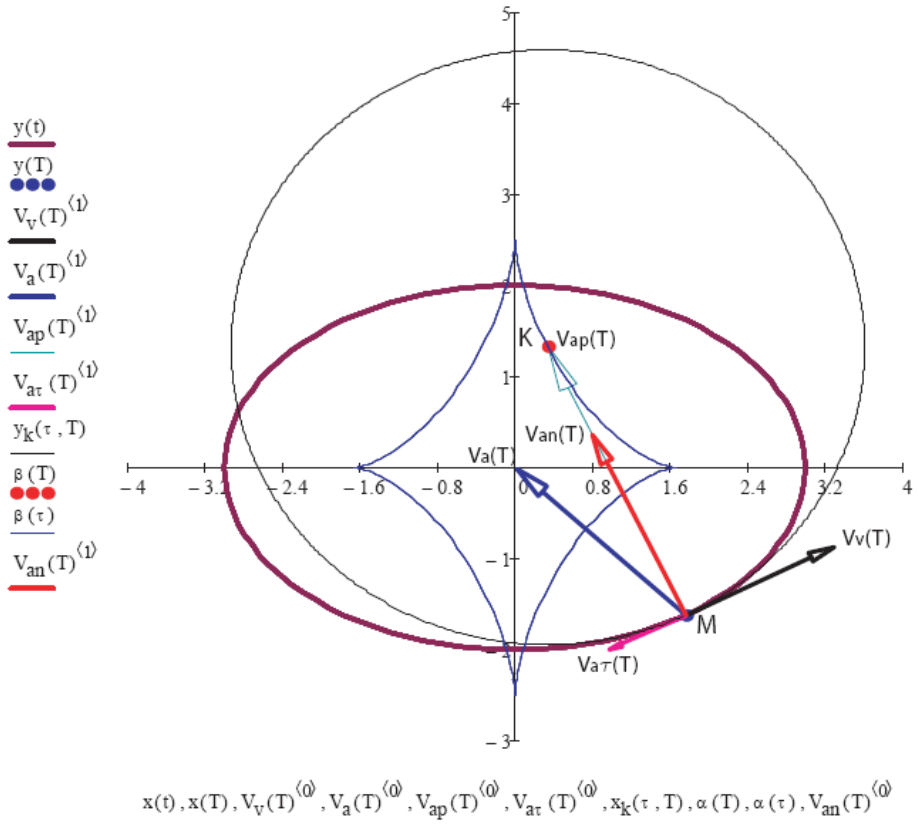
In Fig. 2 these columns draw the vectors shown on the frame of the composed visualized video clip when $T = 1.7$, if in (6) $A = 3$, $B = 2$, $\omega = \pi$. In Fig. 1 the trajectory of the moving point M is a bold curve, the circle of curvature is a thin curve, the curve with corners is the evolute drawn by the centre of curvature K, the bold vector of acceleration is directed to the centre of the co-ordinate system, the bold vector of normal acceleration is directed to the centre of curvature, the bold vector of tangential acceleration is directed opposite the bold vector of velocity (this means that the motion is slowing down at $T = 1.7$), the thin vector connects the point M and the centre of curvature K.

The video clip, with the frame in Fig. 1, clearly shows the change of the point M vector modules of velocity and acceleration in time t. One can also see the motion and change of the radius of the circle of curvature and the creation of the evolute as the trajectory of the centre of curvature.

The composed Mathcad worksheet allows easy modification of Fig. 1. When replacing the law (6) with

$$x(t) = t, y(t) = 0.8 \cdot \sin(3 \cdot t),$$

then the frame 1 in Fig. 1 changes to Fig. 2.



$x(t), x(T), V_v(T)^{\langle 0 \rangle}, V_a(T)^{\langle 0 \rangle}, V_{ap}(T)^{\langle 0 \rangle}, V_{a\tau}(T)^{\langle 0 \rangle}, x_k(\tau, T), \alpha(T), \alpha(\tau), V_{an}(T)^{\langle 0 \rangle}$

y(t)	Trajectory of the point M
y(T)	Point M
Vv(T)	Velocity of the point M
Va(T)	Full acceleration of the point M
Vap(T)	Vector, connecting point M and the centre of curvature
Vaτ(T)	Tangential acceleration of the point
yk(τ,T)	Circle of trajectory's curvature at the point M
β(T)	Centre of trajectory's curvature at the point M
β(τ)	Evolute of the trajectory of the point M
Van(T)	Normal aceleration at the point M

Fig. 1. **Frame from video clip:** http://www.youtube.com/watch?v=zFbVcd4GOJw

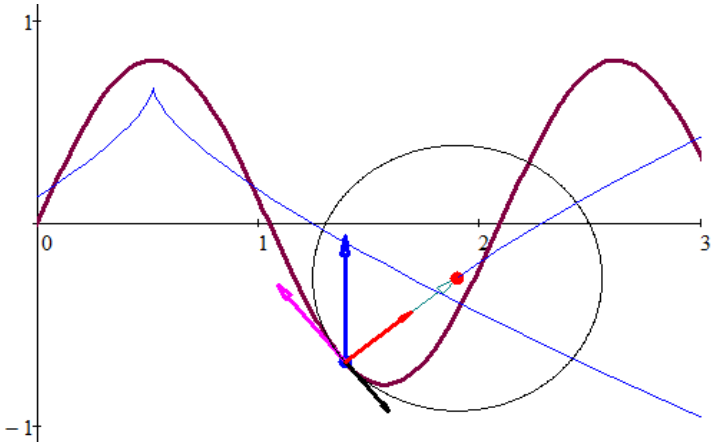


Fig. 2. **A frame of possible second video clip:** http://www.youtube.com/watch?v=2gPSalH9lxU&feature=youtu.be

Conclusions

1. The composed solution on the Mathcad worksheet and already created video clips allow explaining the kinematics of a material point to the students.
2. The environment of Mathcad allows observing how the formulas actually "work".

3. It is important that Mathcad detects most of syntax errors.
4. The possibility of experimentation with lecture materials and composing animations makes the study of engineering subjects interesting and attractive for students.
5. Simple procedure of animation on Mathcad worksheet allows visualization of the problems of engineering subjects for students.

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